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MTEM TO THE (2+1)-DIMENSIONAL ZK EQUATION AND CHAFEE-INFANTE EQUATION

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Abstract. Modified trial equation method (MTEM) was used for exact solutions of (2+1)-dimensional Zakharov-Kuznetsov (ZK) equation and Chafee-Infante equation. Three and two dimensional graphs were plotted to analyze the physical behaviors of the solutions by using Wolfram Mathematica 9.This method is an important method for finding travelling wave solutions of nonlinear partial differential equations (NLPDEs).

Keywords:(2+1)-dimensional ZK equation, Chafee-Infante equation, travelling wave solution, MTEM. **AMS Subject Classification:** 35-04, 35C08, 35N05, 68N15.

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1 Introduction

It is very substantial to find exact solutions of NLPDEs. In recent years, computer programs such as Maple, Matlab that facilitate algebraic calculations and many methods have been used to find the solutions of NLDEs. Several of these methods are $\tan - \tanh(\phi(\xi)/2)$ -expansion method, $\tan(\phi/2)$ -expansion method (Manafian, 2016; Manafian & Heidari, 2019; Ilhan et al., 2020), Semi-Inverse Variational method (Manafian et al. (2020)), Multiple rogue-wave solution method (Lu et al., 2020), (G'/G)-expansion method (Islam & Hasan, 2018), extended trial equation method (Bulut et al., 2014; Demiray et al., 2015), extended sinh-cosh method (Triki& Wazwaz, 2014), sine-cosine method (Wazwaz, 2004; Bibi & Mohyud-Din, 2014), Kudryashov method (Kudryashov, 2012; Pandir et al., 2012), exp-function method (Manafian & Lakestani, 2015; Heris & Zamanpour, 2013), MTEM (Odabasi & Misirli, 2018; Tandogan & Bildik, 2016). In this work, we have applied MTEM to obtain the exact solution of (2+1) dimensional ZK equation and Chafee-Infante equation. By reducing NLPDE to nonlinear ordinary differential equation (NLODE), an algebraic equation system was obtained by Wolfram Mathematica 9. By solving these system, travelling wave solutions have been found. The purpose of this method is to find the traveling wave solutions of NLPDEs. In the light of these data, it is a suitable method to find the solutions of NLPDEs.

Firstly, MTEM is implemented to the following (2+1)-dimensional ZK equation (Khalique & Adem, 2011),

$$u_t + 3hu^2u_x + z(u_{xxt} + u_{xyy}) = 0, (1)$$

where h and z are arbitrary constants. Many methods have been submitted to find the solutions of the Eq. (1) (Naher & Abdullah, 2012; Zhong et al., 2013; Ray, 2018; Alam et al., 2014).

Then, MTEM is applied to the following Chafee-Infante equation Habiba et al. (2019),

$$u_t - u_{xx} - \alpha u(1 - u^2) = 0, \tag{2}$$

where α is arbitrary constant, the parameter α sets the relative equilibrium of the diffusion term and the nonlinear term. Many authors have obtained the solutions of the Chafee-Infante equation using different methods (Straughan, 2020; Huang & Huang, 2017; Qiang et al., 2013).

$\mathbf{2}$ Modified Trial Equation Method

Step 1. Consider the NLPDE,

$$P(u, u_t, u_x, u_{xx}, ...) = 0, (3)$$

wave transform as,

$$u = u(\eta), \eta = kx - ct, \tag{4}$$

where c is a constant. Applying Eq. (4) to Eq.(3), we can observe the following NLODE,

$$O(t, x, u, u', u'', \cdots) = 0, (5)$$

where $u' = \frac{du}{d\eta}$. Step 2. The first order trial equation

$$u' = \frac{S(u)}{R(u)} = \frac{\sum_{i=0}^{n} a_i u^i}{\sum_{j=0}^{l} b_j u^j} = \frac{a_0 + a_1 u + a_2 u^2 + \dots + a_n u^n}{b_0 + b_1 u + b_2 u^2 + \dots + b_l u^l},$$
(6)

and

$$u'' = \frac{S(u) \left[S'(u)R(u) - S(u)R'(u) \right]}{R^3(u)}.$$
(7)

Substituting Eqs. (6) and (7) into Eq.(5), we get

$$q(u) = \chi_0 + \chi_1 u + \ldots + \chi_r u^r = 0.$$
 (8)

Step 3. Equating the coefficients of q(u) to zero, we can obtain

$$\chi_p = 0, p = 0, \dots, r. \tag{9}$$

Solving the system (9), we can find the values of a_0, \ldots, a_n and b_0, \ldots, b_l .

Step 4. Consider Eq.(6), the following integral form can be written

$$\eta - \eta_0 = \int \frac{R(u)}{S(u)} du. \tag{10}$$

Using the complete discrimination system with the roots of S(u), we obtain exact solutions of Eq.(3).

Application to (2+1)-Dimensional ZK Equation 3

Getting transformation as

$$u = u(\eta) = u(x + y - ct), \tag{11}$$

Eq.(1) converts to

$$-cu + hu^3 + z(l-c)u'' = 0.$$
 (12)

By use of balance principle between u'' and u^3 in Eq.(12), we get n = l + 2. Case 1. For l = 0 and n = 2 then

$$u' = \frac{a_0 + a_1 u + a_2 u^2}{b_0},\tag{13}$$

$$u'' = \frac{(a_0 + a_1u + a_2u^2)(a_1 + 2a_2u)}{b_0^2}.$$
(14)

where $a_2 \neq 0$ and $b_0 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$a_0 = a_0, a_1 = 0, a_2 = a_2, b_0 = \mp \sqrt{-\frac{2za_2(ha_0 + a_2)}{h}}, c = -\frac{ha_0}{a_2}.$$
 (15)

Substituting Eq. (15) into Eq. (10), we get the following trigonometric function solution,

$$u(x,y,t) = \sqrt{\frac{a_0}{a_2}} \tan\left[\pm\sqrt{-\frac{ha_0}{2z(ha_0+a_2)}} \left(x+y+\frac{ha_0}{a_2}t\pm\sqrt{-\frac{2za_2(ha_0+a_2)}{h}}\eta_0\right)\right].$$
 (16)

Case 2: For l = 1 and n = 3 then

$$u' = \frac{a_0 + a_1 u + a_2 u^2 + a_3 u^3}{b_0 + b_1 u},$$
(17)

$$u'' = \frac{(a_0 + a_1u + a_2u^2 + a_3u^3)(b_0 + b_1u)(a_1 + 2a_2u + 3a_3u^2) - b_1(a_0 + a_1u + a_2u^2 + a_3u^3)}{(b_0 + b_1u)^3}.$$
(18)

where $a_3 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_{1} = \frac{a_{0}b_{1}}{b_{0}}, a_{2} = -\frac{ha_{0}}{2} + \sqrt{\frac{hza_{0}^{2}(hza_{0}^{2} - 2b_{0}^{2})}{2za_{0}^{2}}},$$

$$a_{3} = \frac{\left(-hza_{0}^{2} + \sqrt{hza_{0}^{2} + (hza_{0}^{2} - 2b_{0}^{2})}\right)b_{1}}{2za_{0}b_{0}}, c = \frac{hza_{0}^{2} + \sqrt{hza_{0}^{2}(hza_{0}^{2} - 2b_{0}^{2})}}{b_{0}^{2}}.$$
(19)

Substituting Eq. (19) into Eq. (10), we have the following dark soliton solution,

$$u_1(x, y, t) = \frac{a_0}{b_0} \sqrt{z\mu} \tanh\left[\frac{1}{\sqrt{z\mu}} \left(x + y - \frac{hza_0^2\mu}{b_0^2} t - 2b_0\sqrt{z\eta_0}\right)\right],$$
(20)

where $\mu = 1 + \sqrt{1 - \frac{2b_0^2}{hza_0^2}}$.

4 Application to Chafee-Infante Equation

Getting transformation as

$$u = u(\eta), \eta = kx - ct, \tag{21}$$

Eq.(2) converts to

$$-cu' - k^2 u'' + \alpha (u^3 - u) = 0.$$
⁽²²⁾

By use of balance principle between u'' and u^3 in Eq. (22), we have n = l + 2. Case 1: For l = 0 and n = 2 then

$$u' = \frac{a_0 + a_1 u + a_2 u^2}{b_0},\tag{23}$$

$$u'' = \frac{(a_0 + a_1u + a_2u^2)(a_1 + 2a_2u)}{b_0^2},$$
(24)

where $a_2 \neq 0$ and $b_0 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$a_0 = 0, a_1 = a_2, c = -\frac{3\alpha b_0}{2a_2}, k = \frac{\alpha b_0^2}{2a_0^2}.$$
 (25)

Substituting Eq. (25) into Eq. (10), we get the following exp-function solution,

$$u_1(x,t) = \frac{\exp\left(\frac{ab_0}{2a_2}\left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}{1 - \exp\left(\frac{ab_0}{2a_2}\left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}.$$
(26)

Case 1.2:

$$a_0 = 0, a_1 = -a_2, c = \frac{3\alpha b_0}{2a_2}, k = \frac{\alpha b_0^2}{2a_0^2}.$$
 (27)

Substituting Eq.(27) into Eq.(10), we have the following exp-function solution,

$$u_2(x,t) = \frac{1}{1 - \exp\left(\frac{ab_0}{2a_2} \left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}.$$
(28)

Case 2: For l = 1 and n = 3 then

$$u' = \frac{a_0 + a_1 u + a_2 u^2 + a_3 u^3}{b_0 + b_1 u},$$
(29)

and

$$u'' = \frac{(a_0 + a_1u + a_2u^2 + a_3u^3)(b_0 + b_1u)(a_1 + 2a_2u + 3a_3u^2) - b_1(a_0 + a_1u + a_2u^2 + a_3u^3)}{(b_0 + b_1u)^3},(30)$$

where $a_3 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_0 = 0, c = -\frac{3k\sqrt{\alpha}}{\sqrt{2}}, a_1 = -a_2, a_3 = 2a_2, b_1 = \frac{2\sqrt{2}ka_2}{\sqrt{\alpha}}, b_0 = -\frac{\sqrt{2}ka_2}{\sqrt{\alpha}}.$$
(31)

Substituting Eq. (31) into Eq. (10), we get the following exp-function solution

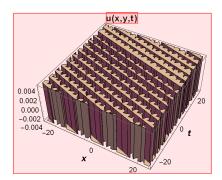
$$u_3(x,t) = \frac{\exp\left(\sqrt{\frac{\alpha}{2}}\left(x + 3\sqrt{\frac{\alpha}{2}}t + \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}{1 - \exp\left(\sqrt{\frac{\alpha}{2}}\left(x + 3\sqrt{\frac{\alpha}{2}}t + \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}.$$
(32)

Case 2.2:

$$a_0 = 0, c = \pm \frac{3k\sqrt{\alpha}}{\sqrt{2}}, a_1 = \mp a_2, a_3 = \pm 2a_2, b_1 = \mp \frac{2\sqrt{2}ka_2}{\sqrt{\alpha}}, b_0 = \frac{\sqrt{2}ka_2}{\sqrt{\alpha}}.$$
 (33)

Substituting Eq. (33) into Eq. (10), we have the following exp-function solution

$$u_4(x,t) = \frac{1}{1 - \exp\left(\mp \sqrt{\frac{\alpha}{2}} \left(x \mp 3\sqrt{\frac{\alpha}{2}}t - \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}.$$
(34)



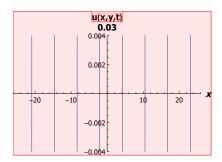
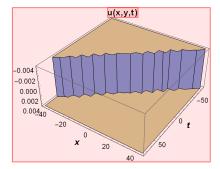


Figure 1: The 3D and 2D surfaces of real values of Eq.(16) for $h = 1, a_0 = 9, a_2 = 3, z = -6, -25 \le x \le 25, -25 \le t \le 25$ and y = 0.02, t = 0.03 for 2D.



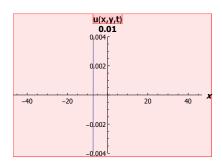
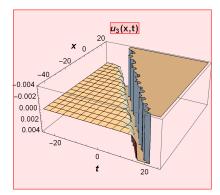
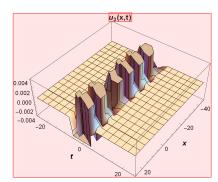


Figure 2: The 3D and 2D surfaces of imaginary values of Eq.(16) for $h = 5, a_0 = 1, a_2 = 7, z = 8, -45 \le x \le 45, -75 \le t \le 75$ and y = 0.5, t = 0.01 for 2D.



0.04 0.004 0.004 0.002 −40 −30 −20 −10 −0.002

Figure 3: The 3D and 2D surfaces of real values of Eq.(32) for $\alpha = 1, -45 \le x \le 25, -25 \le t \le 25$ and t = 0.01 for 2D.



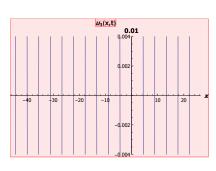


Figure 4: The 3D and 2D surfaces of imaginary values of Eq.(32) for $\alpha = -1, -45 \le x \le 25, -25 \le t \le 25$ and t = 0.01 for 2D.

Remark 1. The solutions of Eq.(1) were procured by using MTEM. These solutions were controlled in Wolfram Mathematica 9. Also, the solutions are new.

Remark 2. The solutions of Eq.(2) were attained by using MTEM. They were checked in Wolfram Mathematica 9. We have attained the similar solution with the solution Eq (3.5) in Habiba et al. (2019) in this study with the solution Eq. (26). Also, other solutions of Eq.(2) are new.

5 Conclusion

In this research, exp-function, dark soliton, trigonometric wave solutions of (2+1)-dimensional ZK equation and Chafee-Infante equation were obtained by using the MTEM. Three and two dimensional graphs for appropriate parameters were plotted to analyze the physical behaviors of the solutions by using Wolfram Mathematica 9. It can be said that MTEM is an effective for finding exact solutions of NLPDEs and it is an important method for obtaining travelling wave solutions. Also, this is a very important method for the solving nonlinear problems.

References

- Alam, Md.N., Akbar, M.A, Fatema, K. & Hafez, G. (2014). Exact traveling wave solutions of the (2+1)-dimensional modified Zakharov Kuznetsov equation via new extended (G'/G)-expansion method. Applied Mathematics, 73, 26267-26276.
- Bibi, S. & Mohyud-Din, S.T. (2014). Traveling wave solutions of KdV using Sine-Cosine Method. Journal of the Association of Arab Universities for Basic and Applied Sciences, 15 (1), 90-93.
- Bulut, H., Pandir, Y. & Tuluce Demiray, S. (2014). Exact Solutions of Nonlinear Schrodinger's Equation With dual power-law Nonlinearity by Extended Trial Equation Method. Waves in Random and Complex Media, 24(4), 439-451.
- Habiba, U., Salam, A., Babul Hossain, Md. & Datta M. Md. (2019). Solitary Wave Solutions of Chafee-Infante Equation and (2+1) Dimensional Breaking Soliton Equation by the Improved Kudryashov Method. *Global Journals*, 19(5), 34-41.
- Heris, J.M. & Zamanpour, I.(2013). Analytical Treatment of the Coupled Higgs Equation and the Maccari System via Exp-Function Method. Acta Universitatis Apulensis, 33, 203-216.
- Huang, H. & Huang, R. (2017). Sign Changing Periodic Solutions for the Chafee–Infante Equation. Applicable Analysis, 97(13),2213-2331.

- Ilhan, O.A., Manafian, J., Alizadeh, A. & Baskonus, H.M. (2020). New exact solutions for nematicons in liquid crystals by the tan(φ/2)-expansion method arising in fluid mechanics. *Eur. Phys. J. Plus*, 135(3), https://doi.org/10.1140/epjp/s13360-020-00296-w, 313.
- Islam, M.M. & Hasan, M.S. (2018). A study on exact solution of the telegraph equation by (G'/G)-expansion method. African Journal of Mathematics and Computer, 11(7), 103-108.
- Khalique, C.M. & Adem, K.R. (2011). Exact solutions of the (2+1)-dimensional Zakharov-Kuznetsov Modified equal width Equation using Lie group analysis. *Mathematical and Computer Modelling*, 54, 184-189.
- Kudryashov, N.A., (2012). One method for finding exact solutions of nonlinear differential equations. Communications in Nonlinear Science and Numerical Simulation, 17(6), 2248-2253.
- Lu, Q., Ilhan, O.A., Manafian, J. & Avazpour, L. (2020). Multiple rogue wave solutions for a variable coefficient Kadomtsev–Petviashvili equation. *International Journal of Computer Mathematics*, https://doi.org/10.1080/00207160.2020.1822996.
- Manafian, J. & Lakestani, M. (2015). Optical Solitons with Biswas-Milovic Equation for Kerr law nonlinearity. *Eur. Phys. J. Plus*, 130(61), 1-12.
- Manafian, J.(2016). Optical soliton solutions for Schrödinger type nonlinear evolution equations by the $\tan(\phi(\xi)/2)$ -expansion method. *Optik*, 127, 4222-4245.
- Manafian, J. & Heidari, S., (2019). Periodic and Singuler Kink Solutions of the Hamiltonian Amplitude Equation. Advanced Mathematical Models & Applications, 4(2), 134-149.
- Manafian, J., Ilhan, O.A., Karmina, K.A. & Mohammed, S.A. (2020). Cross-Kink Wave Solutions and Semi-Inverse Variational Method for (3 + 1)- Dimensional Potential-YTSF Equation. East Asian Journal on Applied Mathematics, 10(3), 549-565.
- Naher, H. & Abdullah, F.A. (2012). The Improved (G'/G)-Expansion Method for the (2+1)-Dimensional Modified Zakharov-Kuznetsov Equation. *Journal of Applied Mathematics*, 2012, Article ID:438928, 1-20.
- Odabasi, M., & Misirli, E. (2018). On the solutions of the nonlinear fractional differential equations via the modified trial equation method. *Mathematical Methods in the Applied Sciences*, 41(3), 904-911.
- Pandir, Y., Gurefe, Y. & Misirli, E. (2012). A new approach to Kudryashov's method for solving some nonlinear physical model. *International Journal of Physical Sciences*, 7(21), 2860-2866.
- Qiang, L., Yun,Z., & Yuanzheng,W. (2013). Qualitative Analysis and Travelling Wave Solution for the Chafee-Infante Equation. *Reports on Mathematical Physics*, 71(2), 177-193.
- Ray, S.S. (2018). Invariant analysis and conservation laws for the time fractional (2+1)dimensional Zakharov-Kuznetsov modified equal width equation using Lie group analysis. *Computers and Mathematics with Applications*, 76, 2110-2118.
- Straughan, B. (2020). Jordan–Cattaneo waves: Analogues of compressible flow. *Wave Motion*, 98, 1-13.
- Tandogan, Y.A., & Bildik, N. (2016). Exact solutions of the time-fractional Fisher equation by using modified trial equation method. AIP Conference Proceedings, 1738, 290018, 1-5.
- Triki, H. & Wazwaz, A.M. (2014). Traveling wave solutions for fifth-order KdV type equations with time-dependent coefficients. *Communications Nonlinear Science and Numerical Simula*tion, 19(3), 404-408.

- Tuluce Demiray, S. Pandir, Y. & Bulut, H. (2015). Solitary Wave Solutions of Maccari System. Ocean Engineering, 103(103), 153-159.
- Wazwaz, A.M. (2004). A Sine-Cosine Method for Handling Nonlinear Wave Equations. Mathematical and Computer Modelling, 40, 499-508.
- Zhong, H., Yu-Feng, Z. & Zhong-Long ,Z. (2013). Double Reduction and Exact Solutions of Zakharov–Kuznetsov Modified Equal width Equation with Power Law Nonlinearity via Conservation Laws. *Communications in Theoretical Physics*, 60(6), 699-706.