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MTEM TO THE (2+1)-DIMENSIONAL ZK EQUATION AND CHAFEE-INFANTE EQUATION

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Abstract. Modified trial equation method (MTEM) was used for exact solutions of (2+1)-dimensional Zakharov-Kuznetsov (ZK) equation and Chafee-Infante equation. Three and two dimensional graphs were plotted to analyze the physical behaviors of the solutions by using Wolfram Mathematica 9.This method is an important method for finding travelling wave solutions of nonlinear partial differential equations (NLPDEs).

Keywords:(2+1)-dimensional ZK equation, Chafee-Infante equation, travelling wave solution, MTEM.
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1 Introduction

It is very substantial to find exact solutions of NLPDEs. In recent years, computer programs such as Maple, Matlab that facilitate algebraic calculations and many methods have been used to find the solutions of NLDEs. Several of these methods are $\tan - \tanh(\phi(\xi)/2)$-expansion method, $\tan(\phi/2)$-expansion method (Manafian, 2016; Manafian & Heidari, 2019; Ilhan et al., 2020), Semi-Inverse Variational method (Manafian et al. (2020)), Multiple rogue-wave solution method (Lu et al., 2020), (G'/G)-expansion method (Islam & Hasan, 2018), extented trial equation method (Bulut et al., 2014; Demiray et al., 2015), extended sinh-cosh method (Triki& Wazwaz, 2014), sine-cosine method (Wazwaz, 2004; Bibi & Mohyud-Din, 2014), Kudryashov method (Kudryashov, 2012; Pandir et al., 2012), exp-function method (Manafian & Lakestani, 2015; Heris & Zamanpour, 2013), MTEM (Odabasi & Misirli, 2018; Tandogan & Bildik, 2016). In this work, we have applied MTEM to obtain the exact solution of (2+1) dimensional ZK equation and Chafee-Infante equation. By reducing NLPDE to nonlinear ordinary differential equation (NLODE), an algebraic equation system was obtained by Wolfram Mathematica 9. By solving these system, travelling wave solutions have been found. The purpose of this method is to find the traveling wave solutions of NLPDEs. In the light of these data, it is a suitable method to find the solutions of NLPDEs.

Firstly, MTEM is implemented to the following (2+1)-dimensional ZK equation (Khalique & Adem, 2011),

$$u_t + 3hu^2u_x + z(u_{xxt} + u_{xyy}) = 0, \tag{1}$$

where h and z are arbitrary constants. Many methods have been submitted to find the solutions of the Eq. (1) (Naher & Abdullah, 2012; Zhong et al., 2013; Ray, 2018; Alam et al., 2014).

Then, MTEM is applied to the following Chafee-Infante equation Habiba et al. (2019),

$$u_t - u_{xx} - \alpha u(1 - u^2) = 0, \tag{2}$$

where α is arbitrary constant, the parameter α sets the relative equilibrium of the diffusion term and the nonlinear term. Many authors have obtained the solutions of the Chafee-Infante equation using different methods (Straughan, 2020; Huang & Huang, 2017; Qiang et al., 2013).

2 Modified Trial Equation Method

Step 1. Consider the NLPDE,

$$P(u, u_t, u_x, u_{xx}, ...) = 0, \tag{3}$$

wave transform as,

$$u = u(\eta), \eta = kx - ct, \tag{4}$$

where c is a constant. Applying Eq. (4) to Eq.(3), we can observe the following NLODE,

$$O(t, x, u, u^{'}, u^{''}, \cdots) = 0, \tag{5}$$

where $u^{'} = \frac{du}{d\eta}$.

Step 2. The first order trial equation

$$u^{'} = \frac{S(u)}{R(u)} = \frac{\sum_{i=0}^{n} a_i u^i}{\sum_{j=0}^{l} b_j u^j} = \frac{a_0 + a_1 u + a_2 u^2 + \ldots + a_n u^n}{b_0 + b_1 u + b_2 u^2 + \ldots + b_l u^l}, \tag{6}$$

and

$$u^{''} = \frac{S(u)\left[S^{'}(u)R(u) - S(u)R^{'}(u)\right]}{R^3(u)}. \tag{7}$$

Substituting Eqs. (6) and (7) into Eq.(5), we get

$$q(u) = \chi_0 + \chi_1 u + \ldots + \chi_r u^r = 0. \tag{8}$$

Step 3. Equating the coefficients of $q(u)$ to zero, we can obtain

$$\chi_p = 0, p = 0, \ldots, r. \tag{9}$$

Solving the system (9), we can find the values of $a_0, \ldots, a_n$ and $b_0, \ldots, b_l$.

Step 4. Consider Eq.(6), the following integral form can be written

$$\eta - \eta_0 = \int \frac{R(u)}{S(u)} du. \tag{10}$$

Using the complete discrimination system with the roots of $S(u)$, we obtain exact solutions of Eq.(3).

3 Application to (2+1)-Dimensional ZK Equation

Getting transformation as

$$u = u(\eta) = u(x + y - ct), \tag{11}$$

Eq.(1) converts to

$$-cu + hu^3 + z(l - c)u^{''} = 0. \tag{12}$$

By use of balance principle between $u^{''}$and u^3 in Eq.(12), we get $n = l + 2$.

Case 1. For $l = 0$ and $n = 2$ then

$$u^{'} = \frac{a_0 + a_1 u + a_2 u^2}{b_0}, \tag{13}$$

$$u^{''}=\frac{(a_0+a_1u+a_2u^2)(a_1+2a_2u)}{b_0^2}. \tag{14}$$

where $a_2\neq 0$ and $b_0\neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$a_0=a_0, a_1=0, a_2=a_2, b_0=\mp\sqrt{-\frac{2za_2(ha_0+a_2)}{h}}, c=-\frac{ha_0}{a_2}. \tag{15}$$

Substituting Eq.(15) into Eq. (10), we get the following trigonometric function solution,

$$u(x,y,t)=\sqrt{\frac{a_0}{a_2}}\tan\left[\pm\sqrt{-\frac{ha_0}{2z(ha_0+a_2)}}\left(x+y+\frac{ha_0}{a_2}t\pm\sqrt{-\frac{2za_2(ha_0+a_2)}{h}}\eta_0\right)\right]. \tag{16}$$

Case 2: For $l=1$ and $n=3$ then

$$u^{'}=\frac{a_0+a_1u+a_2u^2+a_3u^3}{b_0+b_1u}, \tag{17}$$

$$u^{''}=\frac{(a_0+a_1u+a_2u^2+a_3u^3)\,(b_0+b_1u)\,(a_1+2a_2u+3a_3u^2)-b_1(a_0+a_1u+a_2u^2+a_3u^3)}{(b_0+b_1u)^3}. \tag{18}$$

where $a_3\neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_1=\frac{a_0b_1}{b_0}, a_2=-\frac{ha_0}{2}+\sqrt{\frac{hza_0^2(hza_0^2-2b_0^2)}{2za_0^2}},$$

$$a_3=\frac{\left(-hza_0^2+\sqrt{hza_0^2+(hza_0^2-2b_0^2)}\right)b_1}{2za_0b_0}, c=\frac{hza_0^2+\sqrt{hza_0^2(hza_0^2-2b_0^2)}}{b_0^2}. \tag{19}$$

Substituting Eq. (19) into Eq. (10), we have the following dark soliton solution,

$$u_1(x,y,t)=\frac{a_0}{b_0}\sqrt{z\mu}\tanh\left[\frac{1}{\sqrt{z\mu}}\left(x+y-\frac{hza_0^2\mu}{b_0^2}t-2b_0\sqrt{z}\eta_0\right)\right], \tag{20}$$

where $\mu=1+\sqrt{1-\frac{2b_0^2}{hza_0^2}}$.

4 Application to Chafee-Infante Equation

Getting transformation as

$$u=u(\eta), \eta=kx-ct, \tag{21}$$

Eq.(2) converts to

$$-cu^{'}-k^2u^{''}+\alpha(u^3-u)=0. \tag{22}$$

By use of balance principle between $u^{''}$ and u^3 in Eq. (22), we have $n=l+2$.

Case 1: For $l=0$ and $n=2$ then

$$u^{'}=\frac{a_0+a_1u+a_2u^2}{b_0}, \tag{23}$$

$$u'' = \frac{(a_0 + a_1 u + a_2 u^2)(a_1 + 2a_2 u)}{b_0^2}, \tag{24}$$

where $a_2 \neq 0$ and $b_0 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$a_0 = 0, a_1 = a_2, c = -\frac{3\alpha b_0}{2a_2}, k = \frac{\alpha b_0^2}{2a_0^2}. \tag{25}$$

Substituting Eq. (25) into Eq. (10), we get the following exp-function solution,

$$u_1(x,t) = \frac{\exp\left(\frac{ab_0}{2a_2}\left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}{1 - \exp\left(\frac{ab_0}{2a_2}\left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}. \tag{26}$$

Case 1.2:

$$a_0 = 0, a_1 = -a_2, c = \frac{3\alpha b_0}{2a_2}, k = \frac{\alpha b_0^2}{2a_0^2}. \tag{27}$$

Substituting Eq.(27) into Eq.(10), we have the following exp-function solution,

$$u_2(x,t) = \frac{1}{1 - \exp\left(\frac{ab_0}{2a_2}\left[x - \left(-\frac{3a_2}{b_0}\right)t\right] + \eta_0\right)}. \tag{28}$$

Case 2: For $l = 1$ and $n = 3$ then

$$u' = \frac{a_0 + a_1 u + a_2 u^2 + a_3 u^3}{b_0 + b_1 u}, \tag{29}$$

and

$$u'' = \frac{(a_0 + a_1 u + a_2 u^2 + a_3 u^3)\,(b_0 + b_1 u)\,(a_1 + 2a_2 u + 3a_3 u^2) - b_1(a_0 + a_1 u + a_2 u^2 + a_3 u^3)}{(b_0 + b_1 u)^3}, \tag{30}$$

where $a_3 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_0 = 0, c = -\frac{3k\sqrt{\alpha}}{\sqrt{2}}, a_1 = -a_2, a_3 = 2a_2, b_1 = \frac{2\sqrt{2}ka_2}{\sqrt{\alpha}}, b_0 = -\frac{\sqrt{2}ka_2}{\sqrt{\alpha}}. \tag{31}$$

Substituting Eq. (31) into Eq. (10), we get the following exp-function solution

$$u_3(x,t) = \frac{\exp\left(\sqrt{\frac{\alpha}{2}}\left(x + 3\sqrt{\frac{\alpha}{2}}t + \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}{1 - \exp\left(\sqrt{\frac{\alpha}{2}}\left(x + 3\sqrt{\frac{\alpha}{2}}t + \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}. \tag{32}$$

Case 2.2:

$$a_0 = 0, c = \pm\frac{3k\sqrt{\alpha}}{\sqrt{2}}, a_1 = \mp a_2, a_3 = \pm 2a_2, b_1 = \mp\frac{2\sqrt{2}ka_2}{\sqrt{\alpha}}, b_0 = \frac{\sqrt{2}ka_2}{\sqrt{\alpha}}. \tag{33}$$

Substituting Eq. (33) into Eq. (10), we have the following exp-function solution

$$u_4(x,t) = \frac{1}{1 - \exp\left(\mp\sqrt{\frac{\alpha}{2}}\left(x \mp 3\sqrt{\frac{\alpha}{2}}t - \sqrt{\frac{2}{\alpha}}\eta_0\right)\right)}. \tag{34}$$

Figure 1: The 3D and 2D surfaces of real values of Eq.(16) for $h = 1, a_0 = 9, a_2 = 3, z = -6, -25 \leq x \leq 25, -25 \leq t \leq 25$ and $y = 0.02, t = 0.03$ for 2D.

Figure 2: The 3D and 2D surfaces of imaginary values of Eq.(16) for $h = 5, a_0 = 1, a_2 = 7,\ z = 8, -45 \leq x \leq 45, -75 \leq t \leq 75$ and $y = 0.5, t = 0.01$ for 2D.

Figure 3: The 3D and 2D surfaces of real values of Eq.(32) for $\alpha = 1, -45 \leq x \leq 25, -25 \leq t \leq 25$ and $t = 0.01$ for 2D.

Figure 4: The 3D and 2D surfaces of imaginary values of Eq.(32) for $\alpha = -1, -45 \leq x \leq 25, -25 \leq t \leq 25$ and $t = 0.01$ for 2D.

Remark 1. *The solutions of Eq.(1) were procured by using MTEM. These solutions were controlled in Wolfram Mathematica 9. Also, the solutions are new.*

Remark 2. *The solutions of Eq.(2) were attained by using MTEM. They were checked in Wolfram Mathematica 9. We have attained the similar solution with the solution Eq (3.5) in Habiba et al. (2019) in this study with the solution Eq. (26). Also, other solutions of Eq.(2) are new.*

5 Conclusion

In this research, exp-function, dark soliton, trigonometric wave solutions of (2+1)-dimensional ZK equation and Chafee-Infante equation were obtained by using the MTEM. Three and two dimensional graphs for appropriate parameters were plotted to analyze the physical behaviors of the solutions by using Wolfram Mathematica 9. It can be said that MTEM is an effective for finding exact solutions of NLPDEs and it is an important method for obtaining travelling wave solutions. Also, this is a very important method for the solving nonlinear problems.

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